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DYNAMIC ANALYSIS OF THE OUT-OF-PLANE BEHAVIOUR OF MASONRY FAÇADES USING RIGID BEAM MODEL

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Abstract

In this work, a simple and effective Rigid Beam Model originally introduced for studying the dynamic behaviour of ancient freestanding stone columns and recently extended to the case of cantilever unreinforced masonry walls subjected to out-of-plane loading, is further extended to simulate the out-of-plane behaviour of loadbearing façades. Such structural elements are characterized by the presence of at least one or two slabs and roofs, which transfer further vertical loads to the façade and represent additional masses that can be activated by ground acceleration. The proposed model assumes the wall vertically subdivided in equal portions modelled as rigid beam elements and each interface between portions is assumed as a node. Considering no sliding along the interfaces and small displacements of blocks, rocking can be simulated by a bi or tri-linear moment rotation non-linear constitutive law. Monolithic façades with different levels of additional mass on top are modelled and subjected to different in magnitude and frequency harmonic loading. From the analysis results, it is found that monolithic walls can overturn with acceleration magnitudes larger than their corresponding static load multipliers, if input frequency values increase. On the other hand, results converge to static load multipliers for decreasing input frequency values.

Keywords: masonry walls, out-of-plane collapse, Rigid Beam Model, dynamic analysis.

1 INTRODUCTION

The out-of-plane collapse of masonry building façades is a typical mechanism that frequently arises in case of seismic events [1]. As well known, such a mechanism can lead to partial or total collapse of the building, especially in case of inadequate anchorage of façades with diaphragms or orthogonal walls (Figure 1). The analysis of the out-of-plane response of masonry walls is fundamental for the structural assessment of existing buildings, especially the historical ones typical of Italian and south-European town centres. The investigation of this problem is an active field of research in Civil Engineering and Architecture [2].

Freestanding walls or façades in some cases can be assumed as single rigid block structures resting on a foundation. The most important contribution in this field was proposed by Housner [3], who analytically investigated the behavior of moderately slender rigid bodies subjected to horizontal excitations, for estimating the minimum horizontal base acceleration causing the overturning of the system. Many contributions in this field have been proposed up to recent years by means of analytical, numerical and laboratory experimentations, see [4] for further references. In this context, a fast, simple and effective Rigid Beam Model was originally introduced for studying the dynamic behaviour of ancient monolithic and multi-drum freestanding columns [5,6]. This model was recently extended successfully for studying the dynamic out-of-plane behaviour of cantilever monolithic or multi-block masonry walls [4].

In this contribution, the Rigid Beam Model is further extended to simulate the out-of-plane behaviour of loadbearing masonry façades, which support at least one or two slabs and roofs. Such elements transfer further vertical loads to the masonry wall and represent additional masses that can be activated by ground acceleration and can significantly influence out-of-plane collapse mechanisms of the façade. At the same time, they do not represent a further restrain for the façade, especially if the restoration interventions have not been done over the building. It is worth mentioning that in some cases small portions of roofs or slabs transfer loads to adjacent façades, even if they have an inner structure oriented to transfer loads to the orthogonal walls (Figure 1a). This aspect can be also found in a specific monumental masonry building typology, namely churches, where the roof of the building is generally supported by central nave walls [7], but the front portion of the roof can be slightly connected to the façade and can transfer a not negligible load.

The proposed upgraded model follows the typical assumptions taken by Housner [3] and considers each façade portion between two storeys as a monolithic rigid element or subdivided into equally spaced rigid blocks. Each interface between elements or blocks is assumed as a node of the model, which can be characterized by the presence of an additional lumped mass representing a roof or a slab portion supported by the façade. Small displacements and no sliding at interfaces are the main hypotheses of the model, together with a non-linear moment-rotation constitutive law at each interface.

The model effectiveness is evaluated by performing several numerical dynamic tests given by harmonic ground motions with varying input frequency and acceleration magnitude, in order to evaluate the acceleration threshold between safe and collapse conditions.

In this work, a preliminary test campaign is proposed. Numerical analyses focus on monolithic walls having two different values of additional mass on their top, and results are compared with the case without additional loads. Numerical results for decreasing input frequency are also compared with analytical results given by static load multipliers.

The manuscript is organized in three sections. The first one is dedicated to model description detailed to the proposed case study, the second one describes the numerical tests performed and discuss the results obtained, the third one contains final considerations and further developments of the work.

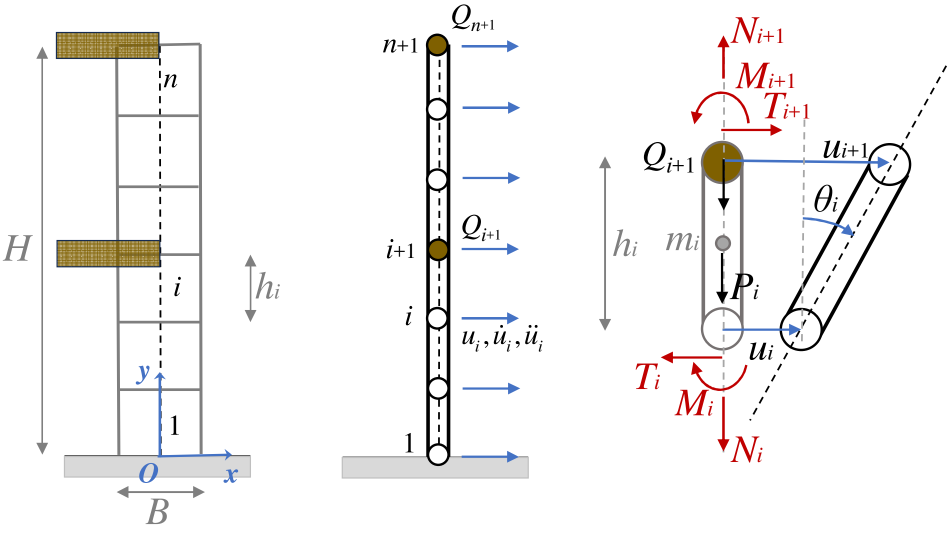
a b

Figure 1: Potential out-of-plane collapse mechanisms of masonry façades with irregular blocks arrangement and not well connected with orthogonal walls [8]. Façade loaded by a portion of the roof (a), façade without additional loads (b).

2 RIGID BEAM MODEL

2.1 Model geometry and kinematics

A freestanding masonry wall or façade subjected to out-of-plane actions is considered. Plane strain conditions are assumed and two-dimensional coordinate system Oxy is introduced. A unitary depth of the wall is then assumed, also neglecting the presence of openings on the façade. The façade is resting on a rigid foundation, without side supports and free to move at its top. Considering its vertical cross-section, it can be subdivided into n layers of equally spaced blocks or portions. Interfaces between the blocks or portions can represent the actual horizontal joints of a regular texture of blocks connected by dry or mortar joints or can represent potential horizontal cracks in case of a wall with an irregular arrangement of blocks [4,8]. The façade has an overall height H and $h_i = H/n$ is the height of the generic i-th block or wall portion (Figure 2a). Wall thickness B is assumed to be uniform along wall height.



a b c

Figure 2: 2D model for a multi-block cantilever wall having uniform width along its height and supporting two generic slabs along its height and top (a), corresponding rigid beam model (b), generic rigid beam element (c).

Nodal horizontal displacements are considered, together with nodal velocities and accelerations (Figure 2b). Each i-th beam element is characterized by a mass m_i, depending on material density γ and on the volume of the corresponding portion. Rigid beam hypothesis allows to define a rigid rotation of each element, depending on the horizontal translations at beam ends and beam height, (Figure 2c): $\theta_i = (u_{i+1} - u_i)/h_i$.
Additional slabs and roofs do not represent further restraints to the façade. For simplicity, they are assumed at wall top (n+1 node) and at the generic i+1 node (Figure 2a) and the corresponding additional loads Q_{i+1}, Q_{n+1} are introduced (Figure 2b).

2.2 Equations of motion

Details of translational and rotational equations of motion of the rigid beam model can be found in the contributions by authors dedicated to multi-drum freestanding columns [5,6], cantilever walls [4], and masonry tall chimneys [9]. If the façade is subjected to a horizontal ground acceleration $a_g(t)$ and the top of the façade is assumed free to move, equations of motion can be written for the entire structure by obtaining the following system of differential equations to be solved:

$$\mathbf{M}(\boldsymbol{\theta}) = \mathbf{G}\mathbf{M}_a\ddot{\boldsymbol{u}} - \mathbf{G}\mathbf{A}_g + \mathbf{I}_G\ddot{\boldsymbol{u}} - \mathbf{B}_g, \tag{1}$$

where $\mathbf{M}$ is a vector collecting bending moments M_i from i to n, depending on the rigid rotations θ_i of the corresponding wall portions. Matrices $\mathbf{M}_a$, $\mathbf{G}$, and $\mathbf{I}_G$ can be called, respectively, mass coefficient matrix, geometric coefficient matrix, and polar inertia coefficient matrix. Details of these matrices, together with vectors $\mathbf{A}_g$ and $\mathbf{B}_g$ can be found in [4]. It must be pointed out that the effect of additional masses is taken into consideration by adding Q_i value to mass matrix $\mathbf{M}_a$ at the corresponding i-th degree of freedom.
The system of differential equations in (1) is solved by means of a Runge-Kutta ODE solver. The nonlinear behaviour of the system is represented by the bending failure at each interface between the wall portions. Following Housner's hypothesis [3], shear failure cannot occur. Considering the approach already adopted by authors, a nonlinear bending moment-rotation relationship at each interface is assumed. This relationship represents the maximum stabilizing moment for varying block rotation and it is slightly modified with respect to Housner's rigid-softening law by means of an initial elastic stiffness depending on masonry elastic modulus and a smoothing parameter [4-6,9]. The maximum stabilizing moment accounts for the maximum eccentricity of the normal force acting at the i-th interface, namely one half of façade width:

$$M_{u,i} = \frac{B}{2}N_i = \frac{B}{2}\left(\sum_{j=i}^{n} P_j + Q_j\right), \tag{2}$$

where $P_j = \gamma\ m_j$ and Q_j is nonzero only when a slab or roof is present. In this work, the contribution of masonry tensile strength is neglected

3 NUMERICAL TESTS

3.1 Main parameters

In this contribution, a preliminary analysis is performed by considering a monolithic façade, hence by assuming n = 1, with additional mass on its top, Q_2. The façade has height H = 5 m, base B = 0.5 m, density γ = 1600 kg/m^3. Considering a unitary façade depth (1 m), these parameters give an overall mass m_1 = 4000 kg/m. Tables 1 and 2 collect two simple examples of gravitational or dead load analyses for two different roof types. A lightweight timber roof can be characterized by a distributed vertical load equal to 158 kg/m^2 (Table 1), whereas a heavy reinforced concrete (or, better, latero-cement) slab can be characterized by a distributed vertical load equal to 328 kg/m^2 (Table 2). Considering in both cases one half of roof span equal to 4 m, additional masses turn out to be equal to 632 kg/m and 1312 kg/m, which represent, respectively, 16% and 32% of façade mass.

roof element	unit. weight [kg/m^2]
roof tiles	60
impermeabilization	10
2-layer wooden floor, 6 cm	36
2 levels of wooden beams	52
total	158

Table 1: Gravitational (dead) load analysis for a lightweight timber roof.

roof element	unit. weight [kg/m^2]
roof tiles	60
impermeabilization	10
latero-cement slab, 20 cm	240
inner plaster, 1.5 cm	18
total	328

Table 2: Gravitational (dead) load analysis for a reinforced concrete roof.

In order to better highlight the effect of the additional mass, the following numerical tests consider the second roof typology (Table 2) by assuming Q_2 equal to the 30% of façade mass, together with an additional heavier roof having Q_2 equal to 50% of façade mass. In the following sub-section, the ratio between additional mass and façade mass is highlighted with parameter $\beta = Q_2/m_1$, which will be assumed equal to 0, 0.3 and 0.5.

3.2 Dynamic analysis, harmonic excitations

In order to evaluate the effectiveness of the update proposal for the Rigid Beam Model, the dynamic behaviour of the façade without and with additional mass on top is evaluated by performing a set of dynamic analyses with harmonic excitations. Results allow to obtain the level of safety and the potential collapse mechanisms of the façades that can be activated by dynamic excitations. Numerical tests are performed by varying input frequency and acceleration magnitude at the base of the façades.

The final results of the campaign of numerical simulations on the monolithic façades are presented in Figure 3 for the case without additional mass, in Figure 4 in case of 30% of additional mass at façade top, in Figure 5 in case of 50% of additional mass. Figures 11-13 show safe (green circles) and unsafe (red crosses) conditions at the end of the harmonic tests for increasing input frequency. Black stepped lines highlight the acceleration threshold that can lead to structural collapse for increasing input frequency. This representation, often defined as 'safe-unsafe domain', was already adopted by authors and it was introduced by Spanos and Koh [10] for rigid blocks. Collapse mechanisms are simply characterized by façade overturning with respect to its base and for brevity they are not shown in this contribution.

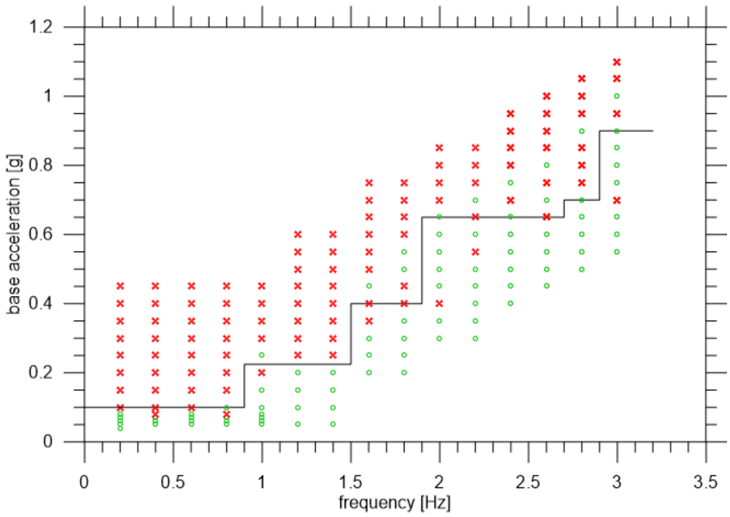


Figure 3: Safe-unsafe domain for the façade subjected to harmonic excitations.

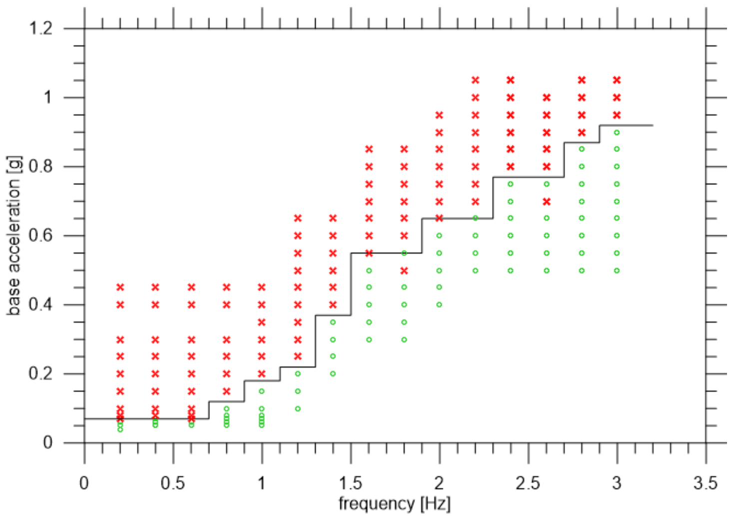


Figure 4: Safe-unsafe domain for the façade with 30% of additional mass on top subjected to harmonic excitations.

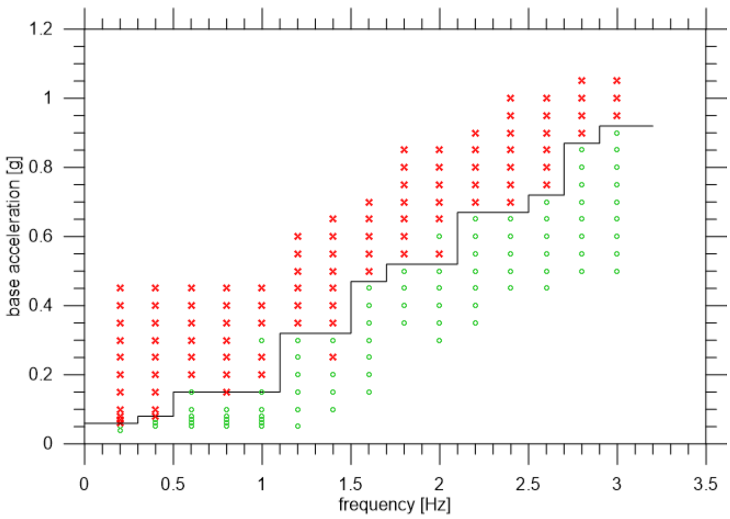


Figure 5: Safe-unsafe domain for the façade with 50% of additional mass on top subjected to harmonic excitations.

It is worth mentioning that for decreasing input frequency results turn out to be in agreement with the static load multipliers of the monolithic façades, without and with additional mass on top:

$$\lambda = (B/H)[(1+\beta)/(1+2\beta)]. \tag{3}$$

The equation above give results equal to 0.10, 0.081, 0.075, in case of β equal to 0, 0.3, 0.5, respectively, representing collapse accelerations equal to 0.1g, 0.081g, 0.075g, respectively. These values are obtained by assuming additional masses acting at façade top mid-section, in agreement with the proposed rigid beam model.
Considering numerical tests results, the collapse acceleration for input frequency tending to zero without additional mass is coincident with the corresponding static load multiplier. In both cases of additional mass, the collapse accelerations obtained for input frequency tending to zero are slightly smaller than the corresponding static load multiplier.
A comparison between the three domains is presented in Figure 6. For increasing input frequency, the acceleration magnitude causing façade collapse increases for all the cases taken into consideration. However, collapse acceleration in case of additional masses are slightly larger than those observed in the case without additional mass. For example, for frequencies between 1.5 Hz and 2.0 Hz, collapse acceleration without additional mass turns out to be close to 0.4g, whereas collapse accelerations for both cases of additional mass are close to 0.5g.

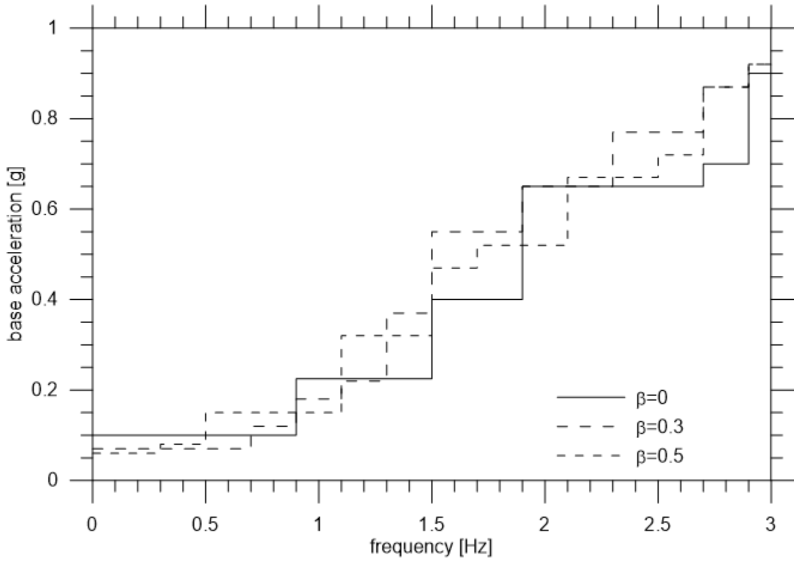


Figure 6: Collapse accelerations for increasing input frequency for the monolithic wall with varying additional mass on its top.

4 CONCLUSIONS

A simple and effective rigid beam model introduced by authors for studying the dynamic behavior of freestanding columns and recently extended to the analysis of cantilever unreinforced masonry walls subjected to out-of-plane dynamic actions has been here further extended by considering additional masses on wall top sections, in order to simulate the dynamic behaviour of masonry building façades loaded by slabs or roofs. In this contribution, a simple case study of a monolithic façade with an additional mass on top was considered and compared to the case without additional mass. Two levels of additional mass were considered, namely 30% and 50% of façade mass. Dynamic analyses of the façades subjected to harmonic ground acceleration with varying input frequency and acceleration magnitude were performed.

Results in terms of collapse acceleration for increasing input frequency were obtained. The following specific comments can be made:

- The proposed model turned out to be fast and effective in performing dynamic analyses and determining safe and unsafe conditions at the end of the numerical tests.
- The effect of additional masses led to smaller collapse accelerations for decreasing input frequencies with respect to the case without additional mass. Collapse accelerations turned out to be in agreement with the static collapse multipliers of the corresponding monolithic façades with and without additional mass.
- Increasing input frequency, collapse accelerations in case of additional mass turned out to be slightly larger than those obtained without additional mass.
- Further developments of this contribution will consider different slenderness values of masonry façades and different ratios between additional mass and façade mass.
- Further developments of this contribution will consider the presence of at least two slabs loading the façade, on one hand by assuming two monolithic portions from façade base to top, on the other hand by assuming more than two subdivisions along façade height, and evaluating the influence of the subsequent multi-drum behaviour in case of additional masses.
- Further developments of this contribution will also focus on dynamic analyses by applying real or scaled ground motions instead of harmonic excitations. This will allow to perform incremental dynamic analyses for generating fragility curves of masonry façades modelled following rigid beam model hypothesis.

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REFERENCES

[1] T.M. Ferreira, A.A. Costa, A. Costa, Analysis of the out-of-plane seismic behavior of unreinforced masonry: a literature review. *International Journal of Architectural Heritage*, **9**(8), 949-972, 2015.

[2] L. Sorrentino, D. D'Ayala, G. Felice, MC Griffith, S. Lagomarsino, G. Magenes. Review of out-of-plane seismic assessment techniques applied to existing masonry buildings. *International Journal of Architectural Heritage*, **11**(1), 2-21, 2016.

[3] G.W. Housner, The behavior of inverted pendulum structures during earthquakes. *Bulletin of Seismological Society of America*, **53**(1), 403-417, 1963.

[4] D. Baraldi, G. Milani, V. Sarhosis, A Rigid Beam Model for studying the dynamic behaviour of cantilever masonry walls. *Structures*, **33**, 2950-2963, 2021.

[5] D. Baraldi, G. Milani, V. Sarhosis, Numerical models for simulating the dynamic behaviour of freestanding ancient columns. *COMPDYN 2019 Proceedings of the 7th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, pp. 1526-1536, Crete island, Greece, June 24-26, 2019.

[6] D. Baraldi, G. Milani, V. Sarhosis, A Simple and Effective Rigid Beam Model for Studying the Dynamic Behaviour of Freestanding Columns. *12th International Conference on Structural Analysis of Historical Constructions SAHC 2020,* Barcelona, Spain (& online event), September 29-October 1, 2021.

[7] G. de Felice, R. Fugger, F. Gobbin, Overturning of the façade in single-nave churches under seismic loading. *Bulletin of Earthquake Engineering*, **20**, 941–962, 2022.

[8] D. Baraldi, E. Reccia, A. Cecchi, Discrete model for out-of-plane loaded random masonry. *COMPDYN 2017 Proceedings of Proceedings of the 6th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering,* pp. 2505-2518, Rhodes island, Greece, June 15-17, 2017.

[9] D. Baraldi, G. Milani, V. Sarhosis, Dynamic Analysis of Masonry Chimneys by Means of a Simple Rigid Beam Model. *COMPDYN 2021 Proceedings of the 8th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, (streamed from) Athens, Greece, June 27-30, 2021.

[10] P.D. Spanos, A.S. Koh, Rocking of rigid blocks due to harmonic shaking. *ASCE Journal of Engineering Mechanics*, **110**(11), 1627-1642, 1984.